Work Function of Strongly Magnetized Neutron Star Crustal Matter and the Associated Magneto-Sphere

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Following an extremely interesting idea [1], published long ago, the work function at the outer crust region of a strongly magnetized neutron star is obtained using relativistic version of Thomas-Fermi type model. In the present scenario, the work function becomes anisotropic; the longitudinal part is an increasing function of magnetic field strength, whereas the transverse part diverges. An approximate estimate of the electron density in the magnetosphere due to field emission and photo emission current, from the polar cap region are obtained.

PACS numbers: 97.60.Jd, 97.60.-s, 75.25.+z

The study of the formation of plasma in a pulsar magnetosphere is a quite old but still an unresolved astrophysical issue. In the formation of magnetosphere plasma, it is generally assumed that there must be an initial high energy electron flux from the magnetized neutron star. At the poles of a neutron star the emitted charged particles flow only along the magnetic field lines. Further a rotating magnetized neutron star generates extremely high electrostatic potential difference near the poles. This potential difference is the driving force and plays the major role in the extraction of electrons as field emission or what is also called cold emission, from the crustal matter of strongly magnetized neutron stars near the poles. The flow of high energy electrons along the direction of magnetic field lines and their penetration through the light cylinder is pictured with the current carrying conductors. Naturally, if the conductor is broken near the pulsar surface the entire potential difference will be developed across a thin gap, called polar gap. This is based on the assumption that above a critical height, from the polar cap, because of high electrical conductivity of the plasma, the electric field $E_{||}$ (= E_0 in this article), parallel with the magnetic field near the poles is quenched. Further, a steady acceleration of electrons originating at the polar region of neutron stars, travelling along the field lines, will produce magnetically convertible curvature γ -rays. If these curvature γ -ray photons have energies $\geq 2m_e c^2$, then pairs of $e^- - e^+$ will be produced in enormous amount with very high efficiency near the polar gap. These produced $e^- - e^+$ pairs form what is known as the magnetospheric plasma [2, 3, 4, 5, 6, 7, 8].

The process of extracting electrons from the outer crust region of strongly magnetized neutron stars, including the most exotic stellar objects, the magnetars, one requires a more or less exact description of the structure of matter in that region [9, 10, 11]. From the knowledge of structural deformation of atoms in strong magnetic field; the departure from spherical nature to a cigar shape, allows us to assume that the atoms in the outer crust region, which are fully ionized because of high density, may be replaced by Wigner-Seitz type cells of approximately cylindrical in structure. We further assume that the electron gas inside the cells are strongly degenerate and are at zero temperature. It is well known that the presence of extraordinarily large magnetic field not only distorts the crystalline structure of dense metallic iron, also significantly modifies the electrical properties of such matter. As for example, the electrical conductivity, which is otherwise isotropic, becomes highly anisotropic in presence of strong quantizing magnetic field. In presence of strong magnetic field iron crystal is highly conducting in the direction parallel to the magnetic field, whereas flow of current in the perpendicular direction is severely inhibited. The aim of this letter is to show that the work function, which is the most important parameter associated with the emission of electrons from the polar region of strongly magnetized neutron stars, will also show anisotropy in presence of strong magnetic field. In this article we have shown that the work function, associated with the emission of electrons along the field lines increases with magnetic field strength. Whereas its transverse component diverges, irrespective of the dimension of the cylindrically deformed atoms. The scenario is very much analogous to the charge transport mechanism in presence of strong magnetic field. To the best of our knowledge, the study of anisotropic nature of work function in presence of strong quantizing magnetic field, which has relevance, specially in the case of magnetized neutron star crustal region has not been studied before. It is also well known that the most important surface emission processes are thermal emission, may be enhanced by Schottky effect, field emission, caused by strong electric field at the poles and perhaps the other important process is the photoelectric emission induced by high energy cosmic photons. The last process of electron emission, if at all possible, has not been studied in this context.

In this letter, following [1] we shall obtain work function associated with the emission of electrons, along and transverse to the direction of magnetic field lines, and show that the later component is infinitely large in this model. In [1] the author have obtained work function related to emission of electrons from cold cathodes. It was associated with some engineering problems. We assume cylindrically deformed atoms in the crustal region with their curved

surfaces parallel to the neutron star surface. Which further means, that locally, the field lines and axes of the cylinders are parallel with each other. To obtain the work function for both the cases, let us consider fig.(1). Here the magnetic field is along z-axis and r-axis is orthogonal to the direction of magnetic field lines. To get the longitudinal part of the work function, we assume that an electron has come out from the cylindrically distorted atom, through one of the plane surfaces, and is at the point P. The electro-static potential at Q produced by this electron is given by

$$\phi^{(e)}(r,z) = \frac{e}{[r^2 + (z-h)^2]^{1/2}}$$
(1)

where AP = h, PD = r and CQ = z. This equation can also be expressed in the following form

$$\phi^{(p)}(r,z) = e \int_{\xi=0}^{\infty} J_0(\xi r) \quad \exp[\mp (z-h)\xi] d\xi$$
for $z > \text{ or } < h$
(2)

For the charge distribution within the distorted atom, the Poisson's equation is given by $\nabla^2 \phi = 4\pi e n_e$ where ϕ is the electro-static potential and n_e is the electron density, given by

$$n_e = \frac{eB}{2\pi^2} \sum_{\nu=0}^{\nu_{max}} (2 - \delta_{\nu 0}) (\mu^2 - m^2 - 2\nu eB)^{1/2}$$
(3)

where ν is the Landau quantum number with ν_{max} the upper limit. The first factor within the sum takes care of singly degenerate 0th Landau level and doubly degenerate other levels with $\nu \neq 0$ and μ is the chemical potential for the electrons, given by $\mu = E_F - e\phi(r) = \text{constant}$, this is the so called Thomas-Fermi condition, E_F is the Fermi energy for the electrons (throughout this article we have taken $\hbar = c = 1$). Hence we can re-write the Poisson's equation in the following form

$$\nabla^2 \phi = \frac{2e^2 B}{\pi} \sum_{\nu=0}^{\nu max} (2 - \delta_{\nu 0}) [(\mu + e\phi)^2 - m_e^2 - 2\nu e B]^{1/2}$$
(4)

To get an analytical solution for ϕ in cylindrical coordinate system (r, θ, z) , with azimuthal symmetry, we put, $m_e = 0$, i.e., the kinetic energy is assumed to be high enough and $\nu_{max} = 0$, which is a valid approximation, provided the magnetic field is too high, so that the electrons occupy only their 0th Landau level. Now defining $\psi = \mu + e\phi$, the solution in the cylindrical coordinate system is given by

$$\psi(r,z) = \int_{\xi=0}^{\infty} J_0(\xi r) a(\xi) = \exp[(\xi^2 + \lambda^2)^{1/2} z] d\xi$$
with $z < 0$ (5)

Here $\lambda^2 = 2e^3B/\pi$ and $a(\xi)$ is some unknown spectral function. Now following [1], we assume that there exist a fictitious secondary field in vacuum, the work function is defined as the work done by this field in pulling out an electron from the material surface to infinity. The secondary field is expressed in coherent with $\phi^{(p)}(r,z)$ and $\phi(r,z)$, and is given by

$$\phi^{(s)}(r,z) = e \int_{\xi=0}^{\infty} J_0(\xi r) f(\xi) \quad \exp(-\xi z) d\xi$$
with $z > 0$ (6)

where $f(\xi)$ is again some unknown spectral function. Then following [1], using the continuity conditions of tangential and transverse components of electric field and the displacement vector respectively, we finally get

$$W_f = \frac{\lambda}{3}e^2 = \frac{1}{3} \left(\frac{2B}{\pi R^{(e)}}\right)^{1/2} m_e e^3 \tag{7}$$

where $B_c^{(e)} \approx 4.43 \times 10^{13}$ G, the critical field strength for electrons to populate their Landau levels in the relativistic scenario. This equation gives the variation of work function with the strength of magnetic field ($\sim B^{1/2}$) associated with the emission of electrons along the direction of magnetic field. From this expression it is also obvious that for a given magnetic field strength, if m_e is replaced by m_p , the proton mass or m_I , mass of the ions, then, since

 $m_I\gg m_p\gg m_e$, the same kind of inequalities are also valid for the work functions associated with the emissions along z-direction. Then obviously, it needs high temperature for thermo-ionic emission of protons or ions, high electric field for their field emissions and high frequency incident photons are essential for the photo-emission of these heavier components.

Let us now consider the emission of electrons in the transverse direction. As shown in fig.(1), p is the position of an electron, came out through the curved surface, then the electro-static potential at q is given by

$$\phi^{(p)}(r,z) = \frac{e}{[z^2 + (r - r_0)^2]^{1/2}}$$
(8)

where $ap = r_0$, cq = r and pd = z. It can also be expressed as

$$\phi^{(p)}(r,z) = e \int_{\xi=0}^{\infty} J_0[\xi(r-r_0)] \exp(\mp z\xi) d\xi$$
for $z > 0$ or $z < 0$ (9)

whereas $\phi(r, z)$ and $\phi^{(s)}(r, z)$ remain unchanged. Then as before, from the continuity conditions on the curved surface, and using some elementary formulas for the Bessel functions, we finally get

$$f(\xi) = \frac{\left[F\left(1 + \frac{\lambda^2}{\xi^2}\right)^{1/2} - G\right]}{\left[1 - \left(1 + \frac{\lambda^2}{\xi^2}\right)^{1/2}\right]}$$
(10)

where $F = J_1[(R-r_0)\xi]/J_1(R\xi)$ and $G = J_0[(R-r_0)\xi]/J_0(R\xi)$. Putting $f(\xi)$ in $\phi^{(s)}(r,z)$ and following [1], evaluating the trivial r_0 -integral and finally using some of the properties of Bessel functions, we have the work function in the case of emission of electrons in the transverse direction

$$W_f = -e^2 \int_{\xi=0}^{\infty} \frac{\left(1 + \frac{\lambda^2}{\xi^2}\right)^{1/2} - \frac{J_1(\xi R)}{J_0(\xi R)}}{1 - \left(1 + \frac{\lambda^2}{\xi^2}\right)^{1/2}} d\xi \tag{11}$$

This is a diverging integral for both $R \longrightarrow \infty$, or $R \longrightarrow 0$, i.e., the value of the work function is independent of the transverse dimension of the cylindrical cell. For $R \longrightarrow 0$, the second term on the numerator vanishes, whereas the rest can be integrated analytically and found to be diverging in the upper limit. On the other hand, for $R \longrightarrow \infty$, the second term becomes $\sim \tan(\xi R - \pi/4)$ (asymptotically). Which itself also diverges for certain fixed values of the argument, but dose not cancel with the infinity from the other part of this integral. Therefore in this limit also the work function becomes infinity. Although we have considered here the extreme case of ultra-strong magnetic field, we do expect that even for low and moderate field values, the work function corresponding to the electron emission in the transverse direction will be several orders of magnitudes larger than the corresponding longitudinal values. In fig.(2) the monotonically increasing straight line curve indicates the variation of W_f with B. This graph clearly shows that for low field values, W_f is a few ev in magnitude, which is of the same orders of magnitudes with the experimentally known values for laboratory metals. However, in this model calculation, since the magnetic field is assumed to be extremely high, we are unable to extend our calculation for very low field values. Also, in this model we can not show the variation of W_f from one kind of metal to another.

As an application of this model we now obtain the density of ultra-relativistic electrons near the poles populated through field emission process. It was first introduced by Fowler and Nordheim [12] to explain cold cathode emission phenomenon. The underline process is the tunneling of metallic electrons through surface potential barrier. Following Fowler and Nordheim we have the electron number density

$$n_f = \frac{\alpha}{2\pi} \frac{E_F^{1/2} E_0^2}{(W_f + E_F) W_f^{1/2}} \exp\left[-\frac{8\pi (2m_e)^{1/2} W_f^{3/2}}{3hc E_0} \right]$$
 (12)

where α is the fine structure constant, E_F is the electron Fermi energy and E_0 is the field at the poles causing electron field emission. In the numerical evaluation of field electron density, we have taken both a constant E_0 (10^{12} V/m), as given in the literature [2] and also as a function of magnetic field strength at the poles, given in [5]. In fig.(3) we have shown the variation of field electron density with the magnetic field strength. Monotonically increasing curves are for magnetic field dependent E_0 . The upper, middle and the lower curves are respectively for $n_e = 10^{-5}$, 10^{-4} and 10^{-3}

times normal nuclear density. Whereas the decreasing curves are for constant E_0 , with the same kind of variation for electron density.

Next, we calculate the photo-electric current in the longitudinal direction. In the relativistic case, in presence of strong quantizing magnetic field at zero temperature the photo-electric current J is given by

$$J = \frac{eB}{2\pi^2} (E_F + E_\gamma - W_f) [1 + \nu_{max}(\nu_{max} + 1)]$$
(13)

where E_{γ} is the incident cosmic ray photon energy. In fig.(2) we have shown the variation of photo-electric current density with the strength of magnetic field. For the sake of illustration, we have taken $E_{\gamma} = 1$ GeV. The variation for $n_e = 10^{-3}$, 10^{-4} and 10^{-5} times normal nuclear density are shown by upper, middle and the lower monotonically decreasing curves respectively. If the photon energy is increased, then it is obvious that J will increase linearly with E_{γ} for constant B and n_e (or E_F).

In conclusion, we mention that the main purpose of this letter is to show that (i) in presence of strong quantizing magnetic field the work function becomes anisotropic, (ii) the transverse part is infinitely large, (iii) the longitudinal part increases with the strength of magnetic field, (iv) low field values of work function are more or less consistent with the tabulated values and finally, (v) the use of longitudinal part of work function gives consistent result associated with magneto sphere electron density, reported in the published literature. In the present work, to the best of our knowledge, the anisotropic nature of work function in presence of strong quantizing magnetic field is predicted for the first time. Further, the diverging character of work function associated with the electron emission in the transverse direction is also obtained for the first time, and so far our knowledge is concerned, it has not been reported in the published work. We have also noticed that in the low magnetic field limit (within the limitation of this model) the numerical values of work function are of the same order of magnitude with the known laboratory data. However, we are not able to compare our result with the variation from one metal to another. As an application the use of magnetic field dependent work function to field emission gives results which are consistent with the published theoretical results.

Since the results presented in this article are very much approximate in nature, we therefore conclude with a bequutiful quotation from a very old but extremely interesting article by Wigner and Bardeen [13]- "It is perhaps not quite superfluous to have, in addition to a more exact calculations of a physical quantity, an approximate treatment which merely shows how the quantity in question is determined, and the lines along which a more exact calculation could be carried out. Such a treatment often leads to a simple formula by means of which the magnitude of the quantity may be readily determined".

Acknowledgement: We are thankful to Professor D. Schieber and Professor L. Schachter of Haifa for providing us an online re-prient of [1].

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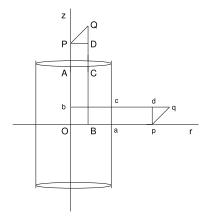


FIG. 1: Schamatic diagram for electron emission along and orthogonal to the direction of magnetic field.

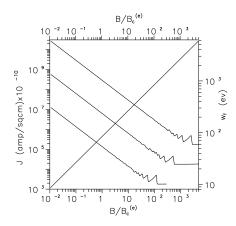


FIG. 2: Variation of work function (W_f) with the magnetic field strength shown by monotonically increasing straight line curve. Variation of photo-electric current with magnetic field. Upper, middle and lower curves are respectively for 10^{-3} , 10^{-4} and 10^{-5} times normal nuclear density.

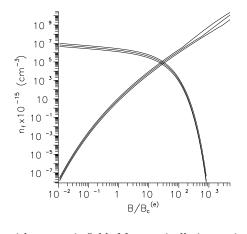


FIG. 3: Variation of field electron density with magnetic field. Monotonically increasing curves are for magnetic field dependent electric field, whereas the set of decreasing curves are for the constant value of electric field as discussed in the text. For both the cases lower, middle and upper curves are respectively for 10^{-3} , 10^{-4} and 10^{-5} times normal nuclear density.